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The turbulent viscosity model and inherent laws of the near-wall turbulence

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Abstract—The model of eddy viscosity published by the present author [*Int. J. Heat Mass Transfer* **26**, 479–508 (1983); **27**, 2415–2420 (1984)] has been applied to the analysis of the conditions in the near-wall region of the pipe flow. On the basis of the coefficients of the model the value of a characteristic property $l^+_{\sigma w} = (du/dr)_w \approx t_{\sigma w} \sim 100$ corresponding to the experimentally determined dimensionless pitch of the horseshoe eddies on the wall in the developed flow was obtained.

THE MODEL of eddy viscosity published by the present author [1, 2] is based on the concept that there are two prevailing stochastic eddy processes in the turbulent transport. The first of them dominates in the turbulent core of the flow, the second one applies in the near-wall region. The present paper supplements the previous publications by considering the phenomena in the near-wall region of the flow. The quantities involved are designated similarly to the previously used [1, 2] nomenclature.

There are three coefficients, A, α and α_w, in the mathematical description of the model of eddy viscosity in a circular pipe:

$$\begin{aligned}\frac{\varepsilon}{\nu} = \frac{2A}{\alpha}\Bigg\{ & Y\left[\exp\left(-\frac{Y^2}{\alpha}\right)-\exp\left(-\frac{Y^2}{\alpha_w}\right)\right] \\ & +(2-Y)\left[\exp\left(-\frac{(2-Y)^2}{\alpha}\right)-\exp\left(-\frac{(2-Y)^2}{\alpha_w}\right)\right] \\ & -(2+Y)\left[\exp\left(-\frac{(2+Y)^2}{\alpha}\right)-\exp\left(-\frac{(2+Y)^2}{\alpha_w}\right)\right] \\ & -(4-Y)\left[\exp\left(-\frac{(4-Y)^2}{\alpha}\right)-\exp\left(-\frac{(4-Y)^2}{\alpha_w}\right)\right]\Bigg\}. \end{aligned} \tag{1}$$

Values of these coefficients taken from ref. [1] are presented in Table 1 and their physical meaning is explained in refs. [1, 2]. The coefficients α and α_w determine, among others, the dimensionless time scales of turbulent processes—Zhukowsky numbers Zh_σ and $Zh_{\sigma w}$:

$$Zh_\sigma = \frac{\nu t_\sigma}{r_w^2} = \sigma_Y^2 = \frac{4-\pi}{4}\alpha \tag{2a}$$

$$Zh_{\sigma w} = \frac{\nu t_{\sigma w}}{r_w^2} = \sigma_{Yw}^2 = \frac{4-\pi}{4}\alpha_w. \tag{2b}$$

Flow in the near-wall region is characterized by Reynolds friction number $Re^* = 2r_w u^*/\nu$, where friction velocity $u^* = \sqrt{\tau_w/\rho} = \sqrt{f/2}\,u_s$ represents the characteristic velocity. When the Reynolds friction number Re^* is formally rewritten into the form of a product of two dimensionless similarity complexes and further reduced to higher terms by the time scale $t_{\sigma w}$ of turbulent phenomena in the near-wall region

$$\begin{aligned} Re^* &= \sqrt{\frac{f}{2}}Re = \frac{2r_w u^*}{\nu} = \frac{2r_w^2}{\nu t_{\sigma w}}\cdot\frac{u^* t_{\sigma w}}{r_w} \\ &= \frac{2}{Zh_{\sigma w}}\cdot\frac{l^*_{\sigma w}}{r_w} = \frac{2}{Zh_{\sigma w}}L^*_{\sigma w} \end{aligned} \tag{3}$$

the first part of the resulting expression is a double of the inverse value of Zhukowsky number—the dimensionless time scale of turbulent processes—and the second part has the meaning of a relative length

$$L^*_{\sigma w} = \frac{l^*_{\sigma w}}{r_w} = Ho^*_{\sigma w} = \sqrt{\frac{f}{2}}Ho_{\sigma w}$$

characterizing the size of turbulent structures in the near-wall region. The stochastic processes in the near-wall region can be therefore characterized by quantity

$$l^*_{\sigma w} = u^* t_{\sigma w} \tag{4}$$

with the dimension of length. This characteristic length can be made dimensionless in the following way

$$\begin{aligned} l^+_{\sigma w} &= \frac{l^*_{\sigma w}u^*}{\nu} = \left(\frac{du}{dr}\right)_w t_{\sigma w} = \frac{f}{4}ReHo_{\sigma w} \\ &= \frac{f}{8}Re^2 Zh_{\sigma w}. \end{aligned} \tag{4'}$$

Calculated values of the dimensionless characteristic $l^+_{\sigma w}$ in the near-wall region are presented in Table 1. For the fully developed turbulent flows the values of $l^+_{\sigma w}$ are slightly higher than 100.

With α_w increasing (i.e. with the Reynolds number decreasing) the values of $l^+_{\sigma w}$ increase moderately. Using the data from ref. [2] the following limiting value of this length at Re_{crit} can be determined

$$\begin{aligned} l^+_{\sigma w,crit} &= \left(\frac{f}{4}Re\right)Ho_\sigma = \frac{48}{(4-\pi)(7-3\sqrt{5})} \\ &= 4\times 47.908 = 191.63. \end{aligned} \tag{5}$$

NOMENCLATURE

A	coefficient of the eddy viscosity model
L	relative length, l/r_w
Y	relative coordinate, y/r_w
Ho	criterion of homochronicity, $u_s t/r_w$
Re	Reynolds number, $2r_w u_s/\nu$
Zh	Zhukowsky number, $\nu t/r_w^2$
Ho^*	friction criterion of homochronicity, $u^* t/r_w$
Re^*	Reynolds friction number, $2r_w u^*/\nu$
f	Fanning friction factor
l	length
r	radius
r_w	pipe radius
s	pitch
s^+	dimensionless pitch, su^*/ν
t	time
u	velocity
u^*	friction velocity, $\sqrt{\tau_w/\rho}$
y	coordinate.

Greek symbols

α, α_w	coefficients of the eddy viscosity model
ε	eddy viscosity
ν	kinematic viscosity
τ_w	wall shear stress
σ^2	dispersion
ρ	density of mass.

Subscripts and superscripts

s	mean
w	related to the wall
σ	corresponding to dispersion
crit	critical
corr	corrected
*	related to friction
+	dimensionless
t	turbulent.

The length scale $l_{\sigma w}^*$ of turbulent structures in the developed turbulent flow has to be substantially lower than the geometric size of the channel. This requirement can be taken as a qualitative criterion in the determination of whether a given turbulent flow is developed or non-developed. For the purpose of comparison, the dimensionless radius of the pipe

$$r_w^+ = \frac{r_w u^*}{\nu} = \sqrt{\frac{f}{2}\frac{Re}{2}} = \frac{Re^*}{2} \tag{6}$$

can be defined. The values of r_w^+ for the individual regimes are presented in Table 1. In the transition from laminar to turbulent flow the dimensionless radius has the value

$$r_{w,crit}^+ = \sqrt{2Re_{crit}} = 67.75 \tag{7}$$

so that $r_w < l_{\sigma w}$.

According to the present opinion the basic processes in the viscous sublayer are formation and decay of

Table 1. Flow characteristics

	α_w			
	10^{-6}	10^{-5}	10^{-4}	10^{-3}
A	1317.8547	416.01126	130.91019	40.833476
A_w	1317.8505	415.99819	130.86907	40.705194
α		$1/\pi =$	0.318309886	
η		0.5		
$(f/4)Re$	828.88808	296.48076	107.66181	40.019964
f	0.002836367	0.003544305	0.004553146	0.006052886
Re	1168946.0	334588.29	94582.349	26446.865
η_w	8.86227×10^{-4}	2.80250×10^{-3}	8.86227×10^{-3}	2.80250×10^{-2}
$Zh_{\sigma w} = \sigma_{Yw}^2$	2.14602×10^{-7}	2.14602×10^{-6}	2.14602×10^{-5}	2.14602×10^{-4}
$l_{\sigma w}^+$	103.97	106.44	109.26	113.57
$(\tau_t/\tau_w)_{max}$	0.9781	0.9615	0.9321	0.8805
$l_{\sigma w,corr}^+$	102.82	104.37	105.48	106.57
$Ho_{\sigma w}$	0.12543	0.35901	1.01429	2.83734
$r_w^+ = Re^*/2$	22010.54	7042.57	2256.42	727.46

$$l_{\sigma w}^* = u^* t_{\sigma w}$$

$$l_{\sigma w}^+ = \frac{l_{\sigma w}^* u^*}{\nu} = \frac{f}{4} Re Ho_{\sigma w} = \frac{f}{8} Re^2 Zh_{\sigma w} = \left(\frac{du}{dr}\right)_w t_{\sigma w}$$

$$l_{\sigma w,corr}^+ = l_{\sigma w}^+ \sqrt{\left(\frac{\tau_t}{\tau_w}\right)_{max}}$$

horseshoe eddies arising on the channel wall in the developed flow with the dimensionless transverse pitch $s^+ = su^*/\nu \sim 100$ [3]. The model of turbulent viscosity gives, therefore, the basic characteristic length of the stochastic processes in the near-wall region which is in accordance with the value of the pitch of horseshow eddies determined experimentally by the visualization technique. As the actual local shear stress decisive for the phenomena in the near-wall region is not known, the friction velocity (wall shear stress) values were considered in the determination of $l^*_{\sigma w}$ [equation (4)]. Better approximation of reality can be obtained taking the maximum turbulent shear stress $\tau_{t,max}$ in the definition of u^* instead of τ_w.

The corrected value of the dimensionless characteristic $l^+_{\sigma w,corr}$ is determined by the expression

$$l^+_{\sigma w,corr} = l^+_{\sigma w}\sqrt{\left(\frac{\tau_t}{\tau_w}\right)_{max}}. \tag{8}$$

The corrected values $l^+_{\sigma w,corr}$ and values of $(\tau_t/\tau_w)_{max}$ are also presented in Table 1.

CONCLUSIONS

Agreement of dimensionless quantity $l^+_{\sigma w}$ of the vortex processes in the near-wall region determined on the basis of the model of turbulent viscosity [1, 2] with the experimentally determined pitch of the horseshoe eddies on the wall, $s^+ \sim 100$, provides indirect evidence that the model reflects the physical nature of the turbulent transport, despite the fact that the model does not explicitly specify this quantity.

REFERENCES

1. J. Šimonek, A model of eddy viscosity and eddy diffusivity of heat, *Int. J. Heat Mass Transfer* **26**, 479–508 (1983).
2. J. Šimonek, Turbulent transport in the transition flow region, *Int. J. Heat Mass Transfer* **27**, 2415–2420 (1984).
3. J. O. Hinze, *Turbulence*, 2nd edn. McGraw-Hill, New York (1975).

LE MODELE DE LA VISCOSITE TURBULENTE ET LOIS INHERENTES DE LA TURBULENCE PARIETALE

Résumé—Le modèle de viscosité turbulente publié par l'auteur [*Int. J. Heat Mass Transfer* **26**, 479–508 (1983); **27**, 2415–2420 (1984)] a été appliqué à l'analyse des conditions dans la région proche de la paroi du tube. A partir des coefficients du modèle, la valeur de la grandeur caractéristique $l^+_{\sigma w} = (du/dr)_w \cdot t_{\sigma w} \sim 100$ correspond aux pas adimensionnel déterminé expérimentalement, des tourbillons en fer à cheval sur la paroi, pour l'écoulement établi.

DAS MODELL DER TURBULENTEN VISKOSITÄT UND DIE GESETZMÄSSIGKEITEN DER WANDNAHEN TURBULENZ

Zusammenfassung—Das vom Autor im *Int. J. Heat Mass Transfer* **26**, 479–508 (1983); **27**, 2415–2420 (1984) veröffentlichte Modell der turbulenten Viskosität wurde zur Analyse der Verhältnisse im wandnahen Bereich der Rohrströmung herangezogen. Auf der Grundlage der Koeffizienten des Modells erhält man den Wert der charakteristischen Eigenschaft $l^+_{\sigma w} = (du/dr)_w t_{\sigma w}$ zu 100. Dies entspricht der experimentell bestimmten dimensionslosen Teilung der "Horseshoe"-Wirbel im Wandbereich bei ausgebildeter Strömung.

МОДЕЛЬ ТУРБУЛЕНТНОЙ ВЯЗКОСТИ И ВНУТРЕННИЕ ЗАКОНОМЕРНОСТИ ПРИСТЕННОЙ ТУРБУЛЕНТНОСТИ

Аннотация—С помощью модели турбулентной вязкости, предложенной ранее автором [международный журнал «Тепло-и массоперенос» **26**, 479–508 (1983); **27**, 2515–2520 (1984)], проведен анализ условий течения в пристенной области. Установлено, что в области развитой конвекции характерная величина $l^+_{\sigma w} = (du/dr)_w \cdot t_{\sigma w} \sim 100$, что соответствует экспериментально полученному безразмерному значению шага подковообразных вихрей на стенке.